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MIXED BOUNDARY VALUE PROBLEMS FOR COMPOSITE TYPE EQUATION

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ABSTRACT

This article presents some mixed problems formulation for third order composite type equation. Under certain conditions on the coefficients and the equation right side, these problems correctness in S. L. Sobolev spaces is proved.

Keywords: mixed boundary value problems, equations of composite type, theorem, cylindrical domain, regular problems, differential equation, S.L. Sobolev spaces, Green's formula.

PROBLEM FORMULATION

Let Ω - an organic simply connected domain in an n-dimensional Euclidean spaceR^n. For simplicity, assume that the (n-1) -dimensional boundary $\partial\Omega$ of Ω belongs to the classC^∞, those in a sufficiently small neighborhood of each point $x^0 = (x_1^0, x_2^0, \dots x_{n)}^0 \epsilon\ \partial\Omega$there is a parametric representation of the surface

$$\partial\Omega : x_j = f(x_1, x_2, \dots., x_{j-1}, x_{j+1,} \dots x_n)$$

$$1 \le j \ge n$$

Such that the function f is infinitely differentiable in this neighborhood.

Assuming that

$$D = (0,1) \times \Omega, S = \partial\Omega \times [0,1]$$

In the cylindrical domain $D \subset R^{n-1}$, we consider the third-order differential equation

$$Lu \equiv -\frac{\partial \Delta u}{\partial t} + k(x,t)U_{tt} + \sum_{\alpha=1}^{n} U_{x\alpha x\alpha} + \alpha(x,t)U_t + \beta(x,t)U + \alpha(t)|U_t|^p U_t = f(x,t) \qquad (1)$$

where,

$$\Delta U = U_{tt} + \sum_{\alpha=1}^{n} U_{x\alpha x\alpha}$$

Note that equations of the form (1) belong to the class of equations of composite type [1], [6]. The study of boundary value problems for equations of composite type is of great interest, especially in the multidimensional case [2]. Throughout, we will assume that

$$K(x,t)\epsilon C(\bar{D}) \cap C^1(\bar{D}), \alpha(x,t)\epsilon C(\bar{D}),$$

$$\beta(x,t)\epsilon C(\bar{D}) \cap C^1(\bar{D}), 0 \le l(t)\epsilon C^1[0,1]$$

$$\text{-}1{-}1 \le \rho < \frac{2}{n-1} \text{at } n \ge 3$$

($p > -1$, arbitrarily at n=1,2).

Denote by $V = (V_x, V_{x1}V_{x2}, \dots V_{xn})$ the internal normal vector to S.

The mixed task. Find a solution to equation (1) in the domain D and such that $\left.\begin{matrix} U(x,t)\,|_{t=1} = 0, U_t\,|_{t=1} = 0 \\ U_{tt}\,|_{t=0} = 0, U(x,t)\,|_{s} = 0 \end{matrix}\right\}$

(2)

Let C_4 denote the class of functions from the space satisfying conditions (2).

$$H = \left\{U : U \epsilon W_2^2(D);\ \frac{\partial \Delta u}{\partial t} \epsilon L_2(D), U_t \epsilon L_p^1(D)\right\}$$

In the space L_p^1 , p=s+2 the norm is defined as follows

$$||U||_{L_p^1(D)}^p = \int_D j(t) \mid U \mid^p dxdt$$

Definition. We call the function u (x, t) a regular solution to problem (1) - (2) if $u(x,t) \epsilon\, C_L$,

$\mid U_t \mid^p U_t \in L_2(D); \frac{\partial}{\partial t}(|U_t|^{3|2} U_t) \in L_2^1(D), \frac{\partial}{\partial xi}(|U_t|^{p|2} U_t) \epsilon L_2^{\gamma}(D)$

and *u (x, t)* satisfies equation (1) almost everywhere in the domain D.

A priori estimates. Mixed problem solvability for composite type equations.

Theorem. Suppose that the above conditions are satisfied for the coefficients of equation (1); let, in addition, everywhere, conditions

$2\alpha(x,t) - \lambda K - K_t - \lambda^2 \geq \delta, \quad -\lambda\beta - \beta_1 \geq \delta, \quad \delta > 0$

$K(x,0) + \lambda \leq 0,\ \beta(x,0) \leq 0$ be satisfied in the domain D

then for any function $f(x,t)$ such that $f \epsilon L_2(D)$ существует there exists a unique regular solution to problem (1)- (2). Let us prove the following estimates:

$$\|U_{tt}\|_{L_2(D)}^2 + \|\nabla U_t\|_0^2 + \|U\|_{W_2^1(D)}^2 + \|U_t\|_{L_p(D)}^p \leq m\|f\|_0^2 (3)$$

$$\left\|\frac{\partial \Delta u}{\partial t}\right\|_0^2 + \sum_{\gamma=1}^m \left\|U_{x\gamma=\gamma}\right\|_0^2 + \frac{\rho+1}{(0{,}5\rho+1)}\left[\left\|\frac{\partial}{\partial t}(|U_t|^{\frac{\rho}{2}} U_t)\right\|_{L_{p(D)}^{\gamma}}^2 + \sum_{N=1}^n \left\|\frac{\partial}{\partial xi}(|U_t|^{\frac{\rho}{2}} U_t)\right\|_{L_{p(D)}^{\gamma}}^2\right] \leq m_2\|f\|_0^2 (4)$$

To do this, we use the Galerkin method (see [3], [4]). Let $\vartheta_n(x,t)$ be the functions of the boundary value problem

$$\Delta\vartheta_n(x,t) = -\mu_n^2 \vartheta_n(x,t) (5)$$

$$\left.\begin{array}{c} \vartheta_n(x,t)|_{t=1} = 0, \quad \vartheta_{nt} - \lambda\vartheta_{nt}|_{t=0} = 0 \\ \vartheta_n(x,t)|_{s=1} = 0 \end{array}\right\} (6)$$

$$U^N(x,t) = \sum_{n=1}^N C_n W_n(x,t)$$

where C_n are determined from a system of nonlinear algebraic equations of the form

$$(LU^N, \vartheta_s)_0 = (f, \vartheta_s)_0, \quad S = 1,2 \dots N \quad (7)$$

The solvability of this system follows from the a priori estimates of approximate solutions obtained below and the acute angle lemma [5]. Multiplying (7) by $2C_s$ and summing over S from 1 to N we obtain the identity

$$(L_U^N, l^{\lambda i} U_l^N)_0 = (f, l^{\lambda i} U_l)_0 \quad (8)$$

Applying the Green formula and the Cauchy inequality to (8)

$$2|a|\cdot|b| \le \gamma|a|^2 + \frac{1}{\gamma}|b|^2, \forall\gamma > 0$$

We get

$$\|U_{\gamma l}\|_0^2 + \|\nabla U_l\|_0^1 + \|U_l^N\|_{l_p(D)}^p + \|U\|_{W_3^1(D)}^2 \le m_3\|f\|_0^2$$

Let us return to the question of the solvability of the system of equations (8).
If you write it in the form $P_N(\vec{c}) = 0$, where

$$\vec{c} = \exp(\lambda t)(C_1, C_2, \dots C_n),$$

That is a fair assessment

$$(P_n(\vec{c}), \vec{c})_{R^N} \ge m_0\|U^N\|_{W_2^1(D)} - m_4$$

where m_0 and m_4 are some positive constants. By virtue of the fact that the linearly envelope $\mathrm{F}(W_1, W_2, \dots W_N)$ is a finite-dimensional space, there exists $R_1(N) > 0$ such that

$$\|U^N\|_{W_2^1(D)} \ge R_1(N)\sum_{k=1}^{N}(C_k)^2$$

Hence the inequality holds

$$(P_n(\vec{c}), \vec{c})_{R^N} \ge R_1(N)|\vec{c}|^2 - m_4 \ge 0,$$

If $|\vec{c}|$ is a large enough quantity, and this is the condition of an “acute angle” sufficient for the solvability of the system of equations (8). In order for the sequence of $\{U^N\}$, solutions to be bounded in (HD), it is necessary to estimate the derivatives $\frac{\partial\Delta 4}{\partial t}$ and U_{xixi}. To this end, thanks to (5)-(7) we can replace ϑ_N в (8) на $-\frac{1}{\mu}\Delta\vartheta_N$. Multiplying (8) by C_n and summing n from 1 to N, we find

$$\left(Lu^N, -\frac{2}{\mu n^2}e^{\lambda t}\left[\frac{\partial\Delta u^N}{\partial x} + 2\lambda U_{tt}^N + \lambda^2 U_t^N\right]\right)_0 = -\frac{2}{\mu_n^2}\left(f, e^{\lambda t}\left[\frac{\partial\Delta u^N}{\partial x} + 2\lambda U_{tt}^N + \lambda^2 U_t^N\right]\right)_0 \quad (9)$$

Using Green's formula, we get

$$\int_D e^{\lambda t}\left\{\left(\frac{\partial \Delta u^N}{\partial x}\right)^2 + \frac{\lambda}{2}\sum_{i=1}^{n}\left(\frac{\partial^2 U^N}{\partial xi^2}\right)^2 - \sum_{i=1}^{N} U_{xixi}^N U_{tt}^N + \frac{\partial \Delta U^N}{\partial t}\right.$$

$$+\left[\left(2\lambda - k(x,t)\right)U_{tt}^N + \left(\lambda^2 + k(x,t)\right)U_t^N + \beta(x,t)U^N\right]$$

$$\left. - \sum_{i=1}^{n}\frac{\partial^2 U^N}{\partial xi^2}\left[\lambda U_{tt}^N + \lambda^2 U_t^N\right] + \left[2\lambda U_{tt}^N + \lambda^2 U_t^N\right] - k(x,t)U_{tt}^N - k(x,t)U_t^N - \beta(x,t)U^N\right\} dxdt$$

$$+\frac{1}{2}\int_S e^{\lambda t}\left[U_{xx}^{N^2} V_1 + \gamma(t)|U_t^N|^{s+1} U_{tt}^N V_1 + \gamma(t)|U_t|^{p+2} U_{xi} V_{xi}\right] dS$$

$$+\int_D e^{\lambda t}\left[l(t)e^{\lambda t}\right]|U_t^N|^{s+1} U_{tt}^N dxdt + 2\int_D e^{\lambda t}\left[l(t) - |U_t^N|^{p+1} U_{tt}^N\right] dD - \lambda^2 \int_D e^{\lambda t}|U_t^N|^{s+2} dD$$

$$+\int_D e^{\lambda t}\gamma(t)(S+1)\left[|U_t^N|^s U_{tt}^{N^2} + |U_t^N|^s U_{xt}^{N^2}\right] dxdt$$

$$=\sum_{i=1}^{6} I_e \qquad (10)$$

(I_1- is the integral over the region, I_2- is the integral over the boundary, I_3, I_4, I_5, I_6- are the integrals over the region with nonlinear terms).

From estimate (3), applying the Cauchy inequality in I_1 we get that the integral I_1 admits the estimate

$$I_1 \geq m_6 \int_D \left[\left(\frac{\partial \Delta u^N}{\partial t}\right)^2 + \sum_{i=1}^{n}\left(\frac{\partial^2 U^N}{\partial xi^2}\right)^2\right] dD \qquad (11)$$

Considering condition (2), we obtain

$$I_2 = \frac{1}{2}\int_0^1 \left[\frac{\partial^2 u^N(x,0)}{\partial xi^2}\right] dx \geq 0$$

The terms from (10) that are not bilinear can be rewritten in the form

$$I_6 = \frac{2(\rho+1)}{(0{,}5\rho+1)}\int_D e^{\lambda t}\gamma(t)\left[\left\{\frac{\partial}{\partial t}\left(|U_t^N|^{p/2} U_t^N\right)\right\}^2 + \sum_{i=1}^{n}\left\{\frac{\partial}{\partial t}\left(|U_t^N|^{p/2} U_t^N\right)\right\}^2\right] dxdt$$

(12)

From the representation (14) it is clear that $I_6 > 0$. It remains for us to evaluate the integrals I_3+ I_4, i.e.

$$I_3 + I_4 = \int_D e^{\lambda t}\left[f(t)e^{\lambda t}\right]|U_t^N|^{s+1} U_{tt}^N dxdt - 2\int_D e^{-\lambda t}\left[\gamma(t)e^{\lambda t}\right]|U_t^N|^{s+1} U_{tt}^N dxdt$$

$$= \int_D e^{-\lambda t}(jt - \lambda j)|U_t^N| U_{tt}^N dxdt$$

By Hölder's inequality, we have

$$I_3 + I_4 \geq -max|\gamma t - \lambda t| \| |U_t^N|^p \|_{L_p(D)} \cdot \|U_t^N\|_{L_p(D)} |U_{tt}^N|_0$$

Where q (as in the Sobolev embedding theorem) is determined from the equality

$$\frac{1}{n} + \frac{1}{q} + \frac{1}{2} = 1$$

Since, according to S$\leq \frac{2}{n-2}$, it follows that $S^n \leq q$ therefore, by (3), we have

$$\| |U_t^N|^p \|_{L_n} \leq \| |U_t^N|^p \|_{L_p} \leq const(\|f\|_0)$$

so,

$$I_3 + I_4 \geq \ -const(\|f\|_0) max|\gamma t - \lambda t| \|U_t^N\|_{L_q} \|U_t^N\|_Q \qquad (13)$$

Under the conditions of the theorem on the parameter S (13), we have the embedding

$$W_2^1(D) C L_q(D) (\text{см}.8)$$

assuming that

$$\text{const}(\|f\|_0) \max|\gamma t - \lambda t| < \delta 3$$

And applying the Cauchy inequalities to (13) we get

$$I_3 + I_4 \geq -\frac{\delta 3}{2} |U_{tt}^N|_0^2 - \frac{\delta 3}{2} |U_t^N|_{W_2^1(D)}^2 \qquad (14)$$

Consequently, the second estimate follows from (10). So we got the necessary a priori estimates (3) - (4) for an approximate solution of (1) - (2) problem. Since all derivatives in equation (1) are quadratically summable over the domain D, it follows from the well-known weak compactness theorem that from an organic sequence of functions $\{U^N\}$ we can extract a weakly convergent subsequence of the function$\{U^{Ni}\}$ such that for $Ni \to \infty$ we have

$U^{Ni} \to U$ weakly в $W_2^2(D)$

$|U_t^{Ni}|^p U_t^{Ni} \to \psi$weakly в$L^j_{\frac{p+2}{p+i}}$

$\frac{\partial}{\partial t}\left(|U_t^{Ni}|^{p/2} U_t^{Ni}\right) \to \zeta_1$ weakly в $L_2^j(D)$

$\frac{\partial}{\partial xi}\left(|U_t^{Ni}|^{p/2} U_t^{Ni}\right) \to \zeta_2$ weakly в $L_2^j(D)$

$\frac{\partial \Delta U^{Ni}}{\partial t} \to \frac{\partial \Delta U}{\partial t}$ weakly в$L_2(D)$

According to Lemma 1.3 of (5, p. 25) on the passage to the limit in a nonlinear term, we have

$$\psi = |U_t|^{p|2} U_t;\ \zeta_2 = \frac{\partial}{\partial t}(|U_t|^{p|2} U_t)$$

$$\zeta_2 = \frac{\partial}{\partial xi}(|U_t|^{p|2} U_t)$$

Now we can make the passage to the limit in identity (10) as $N_i \to \infty$. Thus, the existence of a regular solution, we consider the difference of two possible solutions $V(x,t) = U_1 - U_2$ where U_1 and U_2 are two solutions to the problem, then $V(x,t)$ satisfies the equation

$$LV = -\frac{\partial \Delta V}{\partial t} + K(x,t)V_{tt} + \sum_{i=1}^{n} \frac{\partial^2 V}{\partial xi^2} + \gamma V_t + \beta V + [J|U_{1t}|^p U_{1t} - |U_{2t}|^p U_{2t}]=0 \quad (22)$$

$$\|V_{tt}\|_0^2 + \|V_t\|_0^2 + \|V_{tt}\|_{W_2^1(D)}^2 + \int_D e^{\lambda t} \lfloor |U_{1t}|^p U_{1t} - |U_{2t}|^p U_{2t} \rfloor (U_{1t} - U_{2t}) dD \le 0 \qquad (15)$$

Given the monotonicity of the operator $|U_{1t}|^p U_{1t}$ we obtain

$$\int_D e^{\lambda t} \lfloor |U_{1t}|^p U_{1t} - |U_{2t}|^p U_{2t} \rfloor (U_{1t} - U_{2t}) dxdt \ge 0$$

Then it follows from (15) that $\|V_{tt}\|_0^2 + \|V_t\|_0^2 + \|V\|_{W_2^1}^2 \le 0$ and means V(x,t)≡ 0

e.i. $U_1(x,t) = U_2(x,t)$ в $D. This\ fact\ completes\ the\ proof\ of\ the\ theorem.$

REFERENCES

1. Juraev T.D. Boundary value problems for equations of mixed and mixed-composite types. - Tashkent .: Science, 1979.
2. Kozhanov A.I. The boundary value problem for a class of third-order equations // Report. USSR Academy of Sciences, 1979.V.249, No. 3.C. 536-540.
3. Ladyzhenskaya O.A. Boundary-value problems of mathematical physics. -M.: Science, 1973
4. Larkin N.A. Boundary-value problems as a whole for one class of hyperbolic equations. Sib. Mat. Journal., 1977. T.XVIII, No. 6. -WITH. 1441-1419
5. Lyons J.L. Some methods for solving nonlinear boundary value problems. -M.: Mir., 1972.
6. Salakhitdinov M.S. Equations of mixed-composite type. Tashkent .: Science, 1974.
7. Fletcher K. Numerical methods based on the Galerkin method. M.: Mir, 1998.-S.346
8. Qushimov, B., Ganiev, I. M., Rustamova, I., Haitov, B., & Islam, K. R. (2007). Land degradation by agricultural activities in Central Asia. Climate Change and Terrestrial Carbon Sequestration in Central Asia; Lal, R., Suleimenov, M., Stewart, BA, Hansen, DO, Doraiswamy, P., Eds, 137-146.